

ANALYTICAL ESTIMATE OF CRITICAL HEAT FLUX FOR WATER BOILING  
IN A TUBE

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An approximate equation for the critical heat flux is obtained. The formula is compared with test data on critical heat fluxes at pressures in the range 9.8–196 bar, water mass velocities larger than 4000 kg/m<sup>2</sup>/sec, and superheat temperatures of more than 50°C.

This paper examines the crisis in the bubble boiling regime in forced flow of water in tubes. Experiments [1] have shown that the speed of circulation of the liquid in the wall layer reduces with increase of heat flux and tends to unity when the crisis is reached. From these experiments one can conclude that the boiling crisis is caused by insufficient influx of liquid to the wall from the flowcore. The maximum (critical) heat flux which can be withdrawn from a transverse (or radial) liquid flow is

$$q_{cr} = M_l (i'' - i_l). \quad (1)$$

In boiling of a superheated liquid there are vapor bubbles in the flow. Therefore, only part (a fraction  $1 - \phi_v$ ) of the total volume of the transverse flow is available to the liquid. Taking this into account, Eq. (1) can be written in the form

$$q_{cr} = M(1 - \phi_v)(i'' - i_l). \quad (2)$$

It is known that the transverse (or radial) transport of mass, momentum, and energy in turbulent flow of a single-phase liquid is due to transverse velocity fluctuations. It is natural to postulate that there is an analogy between transport of mass and momentum also in turbulent flow of a two-phase stream.

The transverse (or Reynolds) flux in single-phase flow is [2]

$$M = \frac{\tau_w g}{\omega}. \quad (3)$$

Expressing the shear stress at the wall in terms of the hydraulic friction factor

$$\tau_w = \frac{1}{8} \xi \frac{\rho \omega^2}{g} \quad (4)$$

and substituting into Eqs. (3) and (2), we obtain

$$q_{cr} = \frac{1}{8} \xi \rho \omega (1 - \phi_v)(i'' - i_l). \quad (5)$$

The mean-mixed enthalpy flux is measured (or calculated) in experiments to determine critical heat flux. The liquid enthalpy is

$$i_l = \frac{i - i'' x_t}{1 - x_t}. \quad (6)$$

If the true mass vapor content is expressed in terms of the volume content by the equation

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$$x_t = \frac{\varphi s \frac{\rho''}{\rho'}}{1 - \varphi \left( 1 - s \frac{\rho''}{\rho'} \right)} \quad (7)$$

and substituted into Eqs. (6) and (5), we obtain an expression for the critical heat flux:

$$q_{cr} = \frac{1}{8} \xi \rho \omega r (1-x) \frac{1 - \varphi_V}{1 - \varphi} \left[ 1 - \varphi \left( 1 - s \frac{\rho''}{\rho'} \right) \right]. \quad (8)$$

This equation can be regarded as final if expressions are known for  $\xi$ ,  $\varphi_V$ ,  $\varphi$ , and  $s$ . However, reliable methods of calculating these parameters have not been developed. Therefore, we limit attention to the region of high speeds and superheated liquid, in which the volume vapor content is small. We can expect that the possible errors due to the simplifying assumptions will be small. We make the simplification  $s = 1$  and  $\varphi_V = \varphi$ . According to experiments [3], for boiling in the large superheat region (for  $-0.2 > x\sqrt{\rho'/\rho''} > -2$ ) the hydraulic friction factor is the same as for isothermal flow. We express the friction factor (for simplicity) via the Blasius equation

$$\xi = 0,316 \left( \frac{\mu}{\rho \omega D} \right)^{0,25}. \quad (9)$$

We use the equation [4]

$$\mu = \mu' (1 - \varphi) + \mu'' \varphi \quad (10)$$

to calculate the flow viscosity.

Taking into account the simplifications made, we obtain an approximate expression for the critical heat flux from Eqs. (8), (9), and (10):

$$q_{cr} \simeq 0.04 \rho \omega r (1-x) \left( \frac{\mu'}{\rho \omega D} \right)^{0,25} \left[ 1 - \varphi \left( 1 - \frac{\rho''}{\rho'} \right) \right] \left[ 1 - \varphi \left( 1 - \frac{\mu''}{\mu'} \right) \right]^{0,25}. \quad (11)$$

To check the validity of Eq. (11) we must know the true volume vapor content of the flow. A number of formulas have been proposed for this, but no reliable method has yet been developed. We shall use an empirical relation from [5]:

$$\varphi = \varphi_0 \left( 1 - \frac{x}{x_0} \right)^{1,35}, \quad (12)$$

where

$$\varphi_0 = 1.32 \cdot 10^{-2} q^{0,35} p^{-0,15} (\rho \omega)^{-0,15}; \quad x_0 = -1.8 \cdot 10^{-5} q^{0,7} p^{0,3} (\rho \omega)^{-0,3}.$$

The limits of applications of this formula are as follows:  $p = 5-98$  bar;  $D = 11.7-34.3$  mm;  $\rho \omega = 100-3600$  kg/m<sup>2</sup>/sec;  $q = 0.2-2$  MW/m<sup>2</sup>.

Thus, the system of equations (11) and (12) determines the critical heat flux. From these expressions we can obtain an equation for the boiling crisis, but it turns out that it cannot be solved for  $q_{cr}$ .

To check Eq. (11) we used values of  $q_{cr}$  recommended in [6] for tubes of diameter 8 mm in the range of pressures 49-196 bar, mass velocities 750-5000 kg/m<sup>2</sup>/sec, water superheat 0-75°C, together with test data [7-9], which were obtained for tubes of diameter 2 mm in the range of pressures 9.8-171 bar, mass velocities  $5 \cdot 10^3 - 3 \cdot 10^4$  kg/m<sup>2</sup>/sec, and water superheat 0-225°C. The calculation was carried out using Eqs. (11) and (12) and the method of successive approximations. It was found that for mass velocities of more than 4000-5000 kg/m<sup>2</sup>/sec and water superheat of more than 50-75°C, the calculated values of  $q_{cr}$  agreed with the experimental (Fig. 1) (the mean square deviation is 0.17). This agreement can be regarded as a confirmation that an analogy exists between transfer of mass and momentum during the boiling crisis.

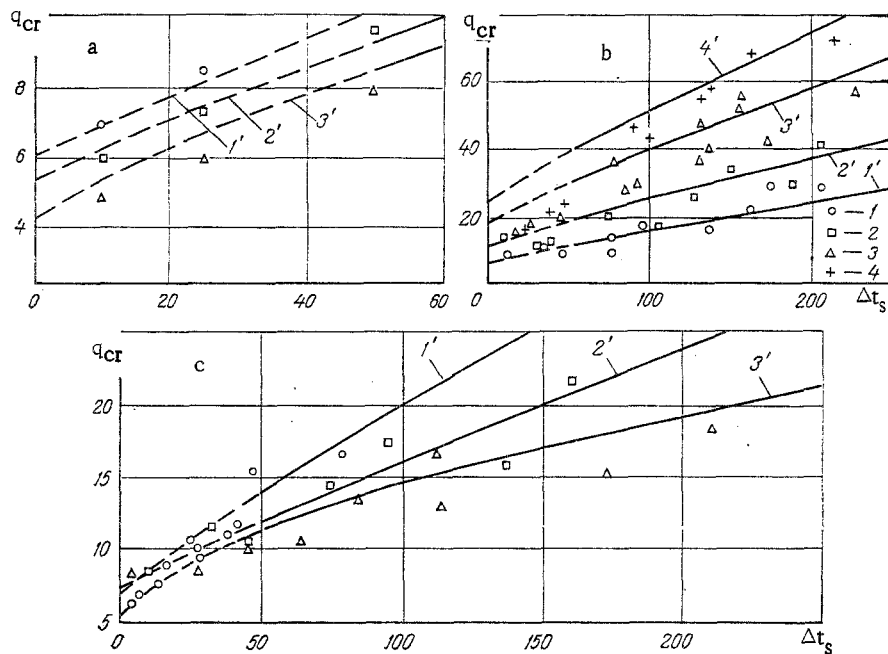


Fig. 1. Comparison of calculated values of  $q_{cr}$  ( $\text{MW}/\text{m}^2$ ) with experiment ( $\Delta t_s$ ,  $^{\circ}\text{C}$ ): a)  $D = 8$  mm;  $\rho w = 5000$   $\text{kg}/\text{m}^2/\text{sec}$ ; 1-3)  $p = 49, 98,$  and  $167$  bar, data of [6]; 1'-3') calculated from Eq. (11); b)  $D = 2$  mm;  $p = 98$  bar; 1-4)  $\rho w = 5 \cdot 10^3, 10^4, 2 \cdot 10^4, 3 \cdot 10^4$   $\text{kg}/\text{m}^2/\text{sec}$ , experiments [7-9]; 1'-4') calculation from Eq. (11); c)  $D = 2$  mm;  $\rho w = 5000$   $\text{kg}/\text{m}^2/\text{sec}$ ; 1-3)  $p = 9.8, 98,$  and  $176$  bar, experiments [6-9]; 1'-3') calculation using Eq. (11).

For low mass velocities (less than  $4000$   $\text{kg}/\text{m}^2/\text{sec}$ ) and low water superheat (less than  $50^{\circ}\text{C}$ ) Eq. (11) underestimates the values of  $q_{cr}$  compared with experiment. A possible reason is that the true volume vapor content values are considerable in this region, and therefore the simplifications made above lead to considerable errors. The analogy between transfer of mass, momentum, and energy in turbulent flow of a single-phase liquid is widely used in calculating heat- and mass-transfer coefficients [2, 10]. There have also been attempts to use the analogy to calculate the boiling crisis. For example, an equation was obtained in [11] for  $q_{cr}$  under forced flow of a vapor-water mixture in tubes. However, because a number of simplifications were made, the agreement between the calculated values of  $q_{cr}$  and the experimental was only to an order of magnitude. An equation was obtained in [12] for  $q_{cr}$  in forced flow of a superheated liquid, but the coefficients entering into the equation were obtained from test data on the boiling crisis.

The discovery of an accurate theoretical equation for the boiling crisis was not one of our objectives. The aim of the work was to check the application of the analogy between transfer of mass and momentum in calculating the boiling crisis. One can hope that by relaxing the simplifications adopted here one might obtain a more accurate equation for the critical heat flux in the future.

#### NOTATION

$D$ , tube diameter, m;  $g$ , acceleration due to gravity,  $\text{m}/\text{sec}^2$ ;  $i$ , mean flow enthalpy,  $\text{J}/\text{kg}$ ;  $i_l$ , heat content of liquid,  $\text{J}/\text{kg}$ ;  $i''$ , heat content of saturated vapor,  $\text{J}/\text{kg}$ ;  $M$ , transverse mass flux (liquid and vapor),  $\text{kg}/\text{m}^2/\text{sec}$ ;  $M_l$ , transverse flux of liquid,  $\text{kg}/\text{m}^2/\text{sec}$ ;  $p$ , pressure, bar;  $p_{cr}$ , critical pressure, bar;  $q$ , heat flux density,  $\text{MW}/\text{m}^2$ ;  $q_{cr}$ , first critical heat flux density,  $\text{MW}/\text{m}^2$ ;  $r$ , heat of vapor formation,  $\text{J}/\text{kg}$ ;  $s$ , coefficient of vapor slip relative to the liquid;  $\Delta t_s$ , superheat of liquid relative to the saturation temperature,  $^{\circ}\text{C}$ ;  $w$ , mean flow velocity,  $\text{m}/\text{sec}$ ;  $x$ , relative balance flow enthalpy;  $x_t$ , true mass vapor content of the flow;  $\mu$ , viscosity of the mixture,  $\text{N}/\text{m}^2/\text{sec}$ ;  $\mu'$ ,  $\mu''$ , viscosity of the liquid and vapor,  $\text{N}/\text{m}^2/\text{sec}$ ;  $\xi$ , hydraulic friction factor;  $\rho w$ , mass velocity of the flow,  $\text{kg}/\text{m}^2/\text{sec}$ ;  $\rho'$  and  $\rho''$ , density of the liquid and vapor,  $\text{kg}/\text{m}^3$ ;  $\tau_w$ , shear stress at the tube wall,  $\text{kg}/\text{m}^2$ ;  $\phi$ , true volume vapor content of the flow;  $\phi_p$ , vapor content of the liquid flowing to the tube wall.

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SOME PROBLEMS OF MEASUREMENT OF THERMAL CONDUCTIVITY BY THE METHOD  
OF COAXIAL CYLINDERS

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An instrument for determining the thermal conductivity of liquids by the method of coaxial cylinders is described. The main attention is devoted to methodological problems of measurements of the thermal conductivity of liquids by this method. Experimental results on the thermal conductivity water obtained using this equipment are presented.

In recent years the method of coaxial cylinders has been extensively used in thermo-physical measurements. Several types of measuring units, operating both in absolute and relative variants, have been made. However, the methodological problems of measuring the thermal conductivity have not been discussed in sufficient detail in many studies. This is largely true in regard to the problems of considering the nonuniformity of the temperature field along the length of the inner cylinder, the temperature distribution in the transverse cross section of the unit, the heat loss, and the radiative component of the heat flux.

The measuring unit described below was developed by us for the investigation of thermal conductivity of liquid solutions in a wide range of variation of the state parameters.

The measuring unit (Fig. 1) was developed according to the absolute method of coaxial cylinders. It was made of refined copper. Its operating surfaces were chrome-plated and polished. A hole of 6 mm diameter was drilled along the axis of the inner cylinder (1), in which an electric heater in a steel Kh18N10T sheath (2) was placed. The heater was a ceramic tube 2 mm in diameter, on which Constantan wire of 0.15 mm diameter was wound; the wire was coated by silk insulation impregnated by high-temperature lacquer. The step of the spiral winding of the heater was 0.35 mm, which made it possible to produce a uniform heat flux in the inner cylinder of the unit. The construction of the heater and the protective sheath almost eliminated unheated segments at the end faces of the cylinder.

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